Geometry I: Triangles

Geometry problems can be difficult to solve. But the problems we have in AMC contests are generally not that difficult. We need to review only a few basic concepts and facts.

Concepts: median, altitude, angle bisector, perpendicular, hypotenuse, interior angle, exterior angle, supplement angles, complement angles, perpendicular bisector.

Simple facts:

1. the sum of all interior angles is 180 degrees.

2. the area of a triangle is equal to one half of the product of the base and the height (altitude).

3. there are three main ways to determine whether two triangles are congruent: SAS, ASA, and SSS.

4. the sum of any two sides is greater than the third side (called the **Triangle Inequality**).

Two most important and used theorems are the following:

One: Pythagorean Theorem: *The square of the hypotenuse of a right-angle triangle is equal to the sum of the squares on the other two sides.*

Two: Similar Triangle Theorem: Two triangles $\triangle ABC$ and $\triangle DEF$ are similar if their corresponding angles are equal, that is, angle A equals angle D, angle B equals angle E, and angle C equals angle F. Similar triangles' sides are proportional, that is, the ratios of the three corresponding sides are equal.

There are other less frequently used theorems as well:

1. **Heron's Formula** Let s be the semi-perimeter of a triangle: $s = \frac{1}{2}(a+b+c)$. Then

$$Area\ of\ the\ trangle = \sqrt{a(s-a)(s-b)(s-c)}.$$

2. **The Angle-bisector Theorem** If D is on side AC of $\triangle ABC$, then,

BD bisects $\angle ABC$ if and only if

$$\frac{AD}{CD} = \frac{AB}{BC}.$$

(The more common version states that if BD bisects the angle B in $\angle ABC$, then $\frac{AD}{CD} = \frac{AB}{BC}$.. It can be proven easily by the similar triangle theorem.)

More advanced theorems:

1. Ceva's theorem: http://en.wikipedia.org/wiki/Ceva's_theorem

2. Perpendicular bisectors of a triangle are concurrent (intersect at a common point).

3. The medians of a triangle are concurrent.

4. The angle bisectors of a triangle are concurrent.

5. The altitudes of a triangle are concurrent.

6. The medians intersect at the **centroid**. The angle bisectors meet at the center of the **inscribed circle** of the triangle. The perpendicular bisectors meet at the center of the **circumscribed circle** of the triangle.