

## Mathematicians and Their Mathematics

*Leonhard Euler (1707 -1783)*

1 Which of the following statements is probably not true about Leonhard Euler's life?

(A) Euler was born in Basel, Switzerland, but spent most his adult life in Russia and Germany.

(B) He was the Director of the Mathematics Class at St. Petersburg Academy during the time of Catherine the Great and died in St. Petersburg.

(C) He wrote many letters to a German Princess explaining his mathematics as well as his views on many other things.

(D) He wrote a letter to George Washington congratulating American people for declaring their independence from the British in 1776.

(E) Euler was a very religious man. As a youth, he was trained to be a pastor.

2 Which of the following statements is probably not true about Euler's mathematical achievements?

(A) Of all the mathematical discoveries of the 18th century in the entire world, about a third belonged to Euler.

(B) He finished his Ph.D. dissertation when he was 20 years old.

(C) He published around 850 mathematical papers in his life.

(D) The constant  $e = 2.7182818284590452354\dots$  was discovered by him and he used the first letter of his last name to denote the constant.

(E) He introduced the concept of a function and was the first to write  $f(x)$  to denote the function  $f$  applied to the argument  $x$ .

3 Which of the following expressions or quantities is not closely related to the irrational number  $e = 2.7182818284590452354\dots$ ?

- (A)  $(1 + \frac{1}{n})^n$ , where  $n$  is a positive integer.
- (B)  $1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!} + \dots$
- (C) The amount of cash (in billions) reported to be raised by the company Google during its Initial Public Offering of its stocks.
- (D)

$$2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4 + \frac{1}{\ddots}}}}}}}}$$

The sequence before the sign  $+$  is  $\{2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, \dots\}$ .

- (E)  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} + \dots$

4 Two numbers are called *Amicable numbers* if the sum of the proper whole number divisors is equal to the other number. For example, the pair 220 and 284. The proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110. The proper divisors of 284 are 1, 2, 4, 71, 142. We have

$$1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284,$$

$$1 + 2 + 4 + 71 + 142 = 220.$$

In the 2000 years before Euler, three such pairs of amicable numbers were found: (220, 284) (ancient Greeks), (17296, 18416) (9th century Islamic mathematician Qurra), and (9363584, 9437056) (17th century Fermat and Descartes).

**Problem** (a) Verify that these two pairs are amicable numbers: (17296, 18416) and (9363584, 9437056).

Euler found 58 other pairs that were published in 1750.

**Problem** (b) Find one of those Euler's amicable numbers.

5 Which of the following formulas was not discovered by Euler?

(A)

$$e^{i\pi} + 1 = 0,$$

where  $e = 2.7182818284590452354 \dots$ ,  $i = \sqrt{-1}$ , and  $\pi = 3.1415926 \dots$ .

(B)

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots + \frac{1}{n^2} + \dots$$

(C)

$$\frac{1}{1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \dots} = (1 - \frac{1}{2^3})(1 - \frac{1}{3^3})(1 - \frac{1}{5^3})(1 - \frac{1}{7^3})(1 - \frac{1}{11^3}) \dots$$

(D)

$$\frac{\pi^{10}}{93555} = 1 + \frac{1}{2^{10}} + \frac{1}{3^{10}} + \frac{1}{4^{10}} + \dots + \frac{1}{n^{10}} + \dots$$

(E)

$$E = mc^2,$$

where  $E$  is the energy,  $m$  is the mass, and  $c$  is the speed of light.

6 Prove that  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{2^n} \geq \frac{n+1}{2}$ , for all positive integer  $n$ .

7 The number  $\ln n$  is defined to be the number such that  $e^{\ln n} = n$ , where  $n > 0$ .

So,  $\ln 1 = 0$  because  $e^0 = 1$ .

$\ln 2$  is approximately 0.69314718 since  $e^{0.69314718} = 1.99999999888 \dots$ .

Euler proved that

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n} - \ln n \rightarrow \gamma,$$

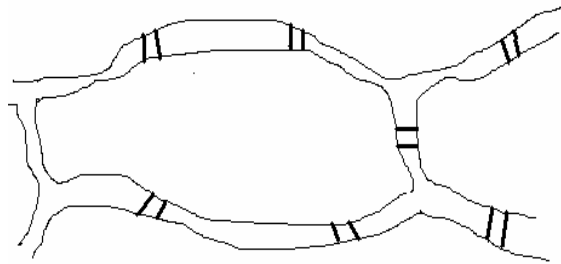
where  $\gamma$  is the Euler constant (or Euler-Mascheroni constant):

$$\gamma = 0.57721566490153286060651209008240243\dots$$

It is still unknown whether  $\gamma$  is a rational or irrational number.

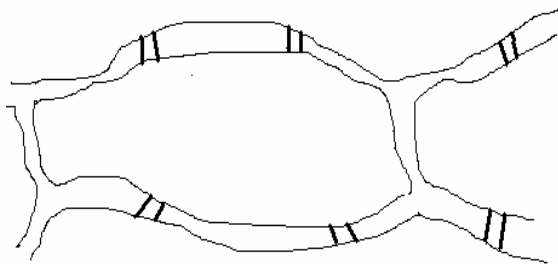
**Problem:** find the first 4 digit prime number in this Euler constant.

8 In Königsberg, Germany, rivers ran through the city. Seven bridges were built so that the people of the city could get from one part to another. A crude map of the center of Königsberg might look like this:



(1) Is it possible for a person to walk around the city in a way that would involve crossing each bridge exactly once?

(2) Suppose that one bridge was collapsed so that the map looked like this:



Now try to solve the same problem.