

Trigonometric facts

1. Values

$$\sin 0 = \sin \pi = \sin 2\pi = \dots = 0.$$

$$\sin \frac{\pi}{2} = 1, \sin \frac{3\pi}{2} = -1.$$

$$\cos 0 = 1, \cos \pi = -1, \dots$$

$$\cos \frac{\pi}{2} \sin \frac{3\pi}{2} = 0.$$

Values of sin and cos of angles of degree 30, 45, 60, 90.

You also try to remember the values of tan and cot.

2. Graphs of $\sin x$ and $\cos x$.

We see from the graph:

$$\sin x = \cos\left(x - \frac{\pi}{2}\right) \text{ or } \sin\left(x + \frac{\pi}{2}\right) = \cos x$$

3. Trigonometric identities.

$$(1) \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y: \text{ (si-co, co-si)}$$

$$(2) \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y: \text{ (co-co, si-si)}$$

More trigonometric identities derived from this pair:

$$(1) \text{ (Pythagorean Theorem) } \sin^2 x + \cos^2 y = 1.$$

$$(2) \text{ (double angle) } \sin 2x = 2 \sin x \cos x, \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x.$$

(3) Half angle formulas are harder to remember. But they can also be easily derived:

Since $\cos 2x = 2 \cos^2 x - 1$, we have $\cos x = 2 \cos^2 \frac{x}{2} - 1$. Or

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}.$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}} \quad \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}.$$