Problems 1, 2 These two problems are easy.

Problems 3, 4 The idea for both problems is the same: need to find the height of the top that is removed from the cone using a pair of similar triangles. The formula you get for Problem 4 should be really nice:

$$V = \frac{h(C_2^3 - C_1^3)}{12\pi(C_2 - C_1)} = \frac{h}{12\pi}(C_2^2 + C_2C_1 + C_1^2).$$

Problem 5 This is a hard problem. You may wish to find the volume of the tip that is cut off. It is a slant pyramid with an ellipse base. The long axis of the ellipse is easy to be found. But the short axis of the ellipse and the height of the slant pyramid are hard to find. You may need to know some trigonometry to tackle this problem.

Problems 6,7,8 use the hints for problems 3 and 4. Note that Problem 8 is not the same as Problem 7: it is harder. In Problem 6, 100 m is the side length.

Problem 9 We discussed how to find the base area: using the Pythagorean theorem. You need to use a couple of more times Pythagorean theorem to find the height of the tetrahedron: connecting the foot of the altitude to one of the vertices of the base.

Problem 10 This is actually easier than it looks: you need to use two pairs of similar triangles to find the side length of the equilateral triangle on the top surface, which is actually 1/3 of 10 cm.

Problem 11 This problem is not an easy one. You assume that these two trapezoids are parallel to each other and are perpendicular to a plane: the ground. You first need to cut the solid on both sides to argue that the volume of it is equal to the volume of a similar solid with rectangular ends. Then, you need to wisely cut this new solid into solids such as prisms, pyramids, whose volumes can be calculated. If you actually use 8 feet as the length, the calculation is quite cumbersome. Changing it to 8 inches can simplify the calculation significantly.

Problem 12 These 5 steps give a good example of how a definite integral, a key concept in calculus, is defined. Be patient with algebra involved. You will be rewarded by discovering the secret of why the formula for the volume of a cone has a factor $\frac{1}{3}$.

Additional Problems from AMC12. AMC12, sometimes, has interesting 3-D geometry problems. We will discuss some of them in the future.

Notes on the last SMM

The mnemonic for the poker game:

The Five Tenacious Boys Nicely Joke to hated servant girls sick for absent Kings.

3 5 10 A J 9 Joker 2 8 7 Q 6 4 (K is absent)

1,2,3 - done by me
4,5,6 - they do the tricks

7,8 - play them straight
We all try 9 for a very long time –
the King, back unseen,
runs up to his Queen,
and takes a final fling to finish the thing!

–J.H. Conway.