Problem 1 This problem is from *Nine Chapters*. But the units are changed. In the book, it stops at $1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{12}$. A harder question we can ask is how many terms we need, if we keep going like this, so that the width will eventually exceed the length.

We can try to estimate how big this number is in terms of n:

$$1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{n}.$$

Search online to find the answer, using the keywords: harmonic sequence, Euler.

Problem 2 This is quite easy. But pay attention to the requirement of the problem: you need to find only the first non-zero digit.

Problem 3 You may wish to use the fact that

$$1^3 = 1, 2^3 = 8, \cdots, 9^3 = 729.$$

Problems 4, 5 There is an error in the first algorithm: In Step 4, first line: *The square root is* $10a + b$ should be *The square root is* $a + b/10 = a.b$.

Use $\sqrt{3}$ to have some practice since we know $\sqrt{3} = 1.732 \cdots$.

It might be a little hard for you to follow this algorithm. Try your best and we will do it on Saturday together.

Problem 6 Problems with * are harder. They are for more advanced students. Try to think why we need to multiply a by 20. What do you have if you multiply out $(10a + b)^2$?.

Problems 7,8 This algorithm is easy to follow. Use fractions in your calculation. Then find the decimal forms. Notice we are getting fractions that are very close to the value of an irrational number $\sqrt{3}$. Can you draw a diagram to explain it? Why $\tilde{a} < \tilde{b}$ when $a < b$?.

Problem 10

If we call

$$x = \cfrac{a}{2 + \cfrac{a}{2 + \cfrac{a}{2 + \cfrac{a}{2 + \cfrac{a}{2 + \cfrac{a}{2 + \cdots}}}}}},$$

do you notice that

$$x = \frac{a}{2 + x}?$$

Problems 11,12 You will find your results are very close to Algorithm 2. Are these two algorithms the same? It is harder for you to explain why it works. But you can try your best.

Others. You may ask why people are interested in different algorithms to find the same thing. The answer is efficiency. We always want to the most efficient way to find a solution. When you are using your calculator, try to think how the calculator works. Which algorithm is it using to find the square root?

There is a famous open problem related to this: *is* **P** *equivalent to* **NP***?* Try to find and understand the exact description of the open problem by searching online using the key words: millennium problems, Clay institute.

Notes on last SMM: For our more advanced students:

1. We know now that the sum must be infinite

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$$

and the sum

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} + \cdots$$

is finite. How about the sum

$$1 + \frac{1}{2^d} + \frac{1}{3^d} \cdots + \frac{1}{n^d} + \cdots$$

for any real number $d > 1$. The answer is that the sum is finite!.

But, of course, the sum depends on d. The bigger d is, the smaller the sum is. This sum

$$z(d) = 1 + \frac{1}{2^d} + \frac{1}{3^d} + \cdots + \frac{1}{n^d} + \cdots$$

is called the Riemann *zeta function*. Search internet to find out more information on this famous function. You will need to know complex numbers to understand its significance.

2. Dr. Carlson asked whether the sum is finite or infinite

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{p_n} + \cdots,$$

where denominators are all prime numbers. What do you think? This has to do with the problem of how many prime numbers are there.

3. Dr. Carlson found a geometric proof of the formula

$$1 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

I have posted it on Blackboard. Hope you will enjoy it.