Problems 1-3. Use the rule stated at the beginning. You will need to find sums like $1 + 2 + \cdots + 9$. This leads to Problem 4.

Problem 4. There are numerous ways to find the formula of the first sum. Try to see how many different ways you can find by yourself. Here is the newest way I learned:

(1) Draw a square of size $n \times n$ units. Count how many squares of 1×1 in this big square in the following way: starting from the lower left corner, going up along the diagonal, count how many squares of size 1×1 in the north and east border. You will have

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

(2) Conclude that $2+4+6+\cdots+(2n) = n^2+n$ and then, find the formula for $1+2+3+\cdots+n$.

Problem 5. Can you draw a trapezoid to show that the formula is valid?

Problems 6-7. Use $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ as the denominator for Problem 6.

Problem 8. This problem actually appeared in the book *Nine Chapters*.

Problem 9. Even though this problem is quite easy to solve, its conclusion is, sometimes, mind boggling. Why?

Problem 10. Even though it is called a geometric progression, I could not find a geometric demonstration for the sum formula. Do you know one?

Problems 11-17. These are just some of typical examples where you can use geometric progression. Problems 11 and 12 are from the book *Nine Chapters*.

Problems 13 and 14 are related to concepts called *fractal images* and *self-similarity*: the graphs consist of parts that are similar to the whole. There are real world examples of such images: coastal lines, fern leaves, snow flakes, etc.

The story in Problem 15 was made famous by Mao. It became very popular during the Culture Revolution in China between 1966 and 1976. The answer to this problem is 19 generations.

Problems 16 and 17 are slightly more complicated applications. You need to make an assumption on when the money is deposited in each month. Assume it is deposited at the beginning of each month. The solution to Problem 16 is \$23,2175.55. For Problem 17, the monthly payment is \$603.65 with the last payment \$603.61. If you know how to use Microsoft Excel, these two problem can be solved easily by using a recursive relation.

Additional Problems. Using Pascal's triangle, one can find the formula of the sum of squares of the first n whole numbers. In fact, one can find the formula of the sum of kth powers of the first n whole numbers using Pascal's triangle. The triangle was studied by B. Pascal, although it had been described centuries earlier by Chinese mathematician Yanghui (about 500 years earlier) who was the other main commentator of the book *Nine Chapters*. It is known as the Yanghui triangle in China. Go to http://ptri1.tripod.com/ to find more fascinating facts about the triangle.