Chapter 7

Excess and Deficit

2000 years ago, people did not have the notation of algebra. They did not even have the negative numbers. But the problems they considered were very similar to the ones we have today. This chapter discusses how to solve certain type of systems of linear equations of two variables. But the concept of equations will not be introduced until next chapter. This chapter contains 20 problems. The first 8 problems explain how the rules work. The rest of the problems are exercises. The main method for solving these problems is the *Rule of Double False*. Again the context of some of the problems are changed. Try to use both algebraic and non-algebraic methods to solve these problems.

Problem 1. A group of people went to a restaurant. After the meal, they decided to divide the bill evenly among themselves. They found out if each paid 8 dollars, the excess was 3 dollars. If each paid 7, it was 4 dollars short (the deficit is 4). Tell: the number of people and the total cost of the meal.

Problem 2. Now chickens are purchased jointly by a group of people. If each contributes \$9, the excess is \$11. If each contributes \$6, the deficit is \$16. How many people are there and how much do the chickens cost?

Problem 3. A group of people go to an antique market to buy jade. If everyone contributes \$ $\frac{1}{2}$, the excess is \$4. If everyone contributes \$ $\frac{1}{3}$, the deficit is \$3. How many people are there in the group and how much does the jade cost?

Problem 4. Now cattle are purchased jointly. If every 7-household contribute 190, the deficit is \$330. If every 9-household contributes 270, the excess is \$30. How many households are there and how much do cattle cost?

Problem 5. Now Gold is purchased jointly. If everyone contributes \$400, the excess is \$3400. If If everyone contributes \$300, the excess is \$100. How many people are there and how much does the Gold cost?

Problem 6. Now sheep are purchased jointly. If everyone contributes \$5, the deficit is \$45. If If everyone contributes \$7, the deficit is \$3. How many people are there and how much do the sheep cost?

Problem 7. Now pigs are purchased jointly. If everyone contributes \$100, the deficit is \$100. If If everyone contributes \$90, it is exactly the right amount. How many people are there and how much do the pigs cost?

Problem 8. Now dogs are purchased jointly. If everyone contributes \$5, the deficit is \$90. If If everyone contributes \$50, it is exactly the right amount. How many people are there and how much do the dogs cost?

After these 8 problems, you may wonder it is possible to design a problem so that no solutions can be found? The answer is yes. We can come up with contradictory information so that solutions do not exist.

For example, let us change Problem 6 to the following:

Problem 6'. Now sheep are purchased jointly. If everyone contributes \$5, it is \$45 short of buying 3 sheep. If everyone contributes \$10, it is \$80 short of buying 6 sheep. Find the number of people in the group and the cost of each sheep.

We know immediately that it is impossible to have a solution since if everyone doubles the contribution to buy twice as many sheep, the deficit should be doubled too.

In algebra, this corresponds to a (non-homogenous) linear system that does not have a solution:

$$\begin{cases} ax + by = c \\ Ax + By = C. \end{cases}$$

If (A, B) is proportional to (a, b), then C must be proportional to c with the same ratio in order for a solution to exist.

But if we change \$80 to \$90, we will have many possible solutions (infinitely many, if fractions are allowed.)

The mathematics that deals with this type of problems is called *Linear Algebra*.

Try the following problem, which is not from the *Nine Chapters*. But it is a natural extension of these problems. Similar problems will be discussed in the next chapter.

Problem 9. Now chickens and pigs are purchased jointly. If everyone contributes \$10 to buy 10 chickens and 4 pigs, the deficit is \$10. If everyone contributes \$12, to buy 5 chickens and 5 pigs, the deficit is \$1. If every one contributes \$15, they can buy exactly 15 chickens and 5 pigs. Find out how many people in the group and how much each chicken and each pig cost.

What is interesting is that the Rule of Double False can be used to solve some problems we have already encountered.

Problem 10 . There is a wall of 9 feet. A gourd is planted above and its vine is creeping down 8 inches a day. A calabash is planted below and its vine is creeping up 1 foot a day. Find the number of days till they meet.

Method: Assume 5 days, the deficit is 8 inches short. Assume 6 days, the excess is 1 foot.

Problem 11. A wall is 5 meters thick. two rats tunnel from opposite sides. On the first day both the big and small rats tunnel a meter each. The big rat doubles its rate daily while the small rat halves its rate daily. Find the number of days till the two rats meet.

Problem 12. 5 large containers and 1 small container have a total capacity of 3 gallons. 1 large container and 5 small containers have a total capacity of 2 gallons. What is the capacity of each container?

Problems like these are numerous. From solving these problems, we can also appreciate the power of algebra.

Problem 13. There are certain number of chickens and rabbits in a cage. At the feeding time, one counts that there are total 35 heads. When one counts legs, there are 96. How many chickens and rabbits are there?

Some of you may have heard about the Chinese Remainder Theorem. Even though the problems sound similar, but they are solved very differently. The Chinese Remainder Theorem appeared in literature later than the *Nine Chapters*. Try to develop an algorithm to find the solutions of the next two problems.

Problem 14. Find the smallest positive integer x such that when it is divided by 8 the remainder is 6 and when it is divided by 9 the remainder is 7.

Problem 15. Find the smallest positive integer x such that when it is divided by 5 the remainder is 3 and when it is divided by 13 the remainder is 7.